QQI
Quality and Qualifications Ireland
Dearbhú Cáilíochta agus Cáilíochtaí Éireann

Special Purpose Specification NFQ Level 5

Maths for STEM 5S2246

1. Component Details

Title	Maths for STEM
Teideal as Gaeilge	.
Award Type	Minor
Code	5S2246
Level	5
Credit Value	30
Purpose	This award specification is designed to facilitate access to higher education STEM (Science, Technology, Engineering and Mathematics) programmes subject to the agreement of the HE institutions concerned. The award will be available to those learners who have demonstrated knowledge, skill and competence in mathematics suitable for successful participation in HE STEM programmes (as prescribed in this award specification). This award may be used for access to higher education and/or in place of the 15 credit L5 Math minor (5N1833) where it appears in certificate requirements for a major award. If desired the additional 15 credits from the Math for STEM award may be applied to the pool of residual credit or in place of any mathematically oriented minor in the elective pool of the same major award
Learning Outcomes	Learners will be able to: The learning outcomes are grouped into the following units starting with an overarching unit setting out expected learning outcomes that apply across all units.

1. Mathematical Proficiency
2. Number
3. Set Theory and Logic
4. Algebra
5. Functions and Calculus
6. Geometry and Trigonometry
7. Probability and Statistics

A brief introduction and purpose statement is provided with each unit.

1 Mathematical Proficiency

The purpose of the outcomes presented in this unit (Mathematical Proficiency) is to recognise learners who are mathematically proficient.

The expected learning outcomes in units 2-7 set out the scope of the expected mathematical proficiency. These express outcomes in units that cover parts of mathematics, for example number, but this must not be interpreted as an expectation that learners will learn the parts in a disconnected way without reference to the other parts.

Learners must not only be able to demonstrate procedural fluency but the broader mathematical proficiency required to solve unfamiliar problems within the scope of this specification. This proficiency includes the ability to abstract mathematical models of certain kinds of physical entities, perform calculations using them and make inferences about the physical entities on the basis of these calculations.

In the language of 'Project Maths[1]' students need to 'mathematically proficient'. Mathematical proficiency is characterised by 5 components:

- **Conceptual understanding:** Comprehension of mathematical concepts, operations and relations
- **Strategic Competence:** The ability to formulate, represent and solve mathematical problems
- **Procedural fluency:** Skill in carrying out procedures flexibly, accurately, efficiently and appropriately
- **Adaptive reasoning:** Capacity for logical thought, reflection, explanation and justification
- **Productive disposition:** Habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy.

[1] http://www.projectmaths.ie/

2 Number

Number is a key concept in science, technology, engineering and mathematics (STEM). A strong knowledge of, and skills in, basic mathematical calculation, and competence to apply these with mastery, is essential for successful participation in STEM programmes.

The purpose of the outcomes presented in this unit (Number) is to recognise learners who have an insight into the use and application of numbers and numerical operations and have mastered the skills for reliable and accurate calculation.

Achievement (with mastery) of the learning outcomes in this unit is essential for award of the L5 Maths for STEM qualification.

2.1 Master the operations of addition, multiplication, subtraction and division in the N, Z, Q, R, domains. Represent these numbers on a number line. Understand absolute value as a measure of distance on the number line.

2.2 Be able to make basic calculations without any errors, with and without the use of a calculator. Verify the accuracy of these calculations using estimates and approximations.

2.3 Convert fractions to percentages, and numbers to scientific notation and calculate percentage error.

2.4 Solve practical problems by choosing the correct formula(e) to calculate the area and perimeter of a square, rectangle, triangle, and circle, giving the answer in the correct form and using the correct units.

2.5 Solve practical problems by choosing the correct formula(e), to calculate the volume/capacity and surface area of a cube, cylinder, cone, and sphere, giving the answer in the correct form and using the correct terminology

2.6 Use the trapezoidal rule to approximate area.

2.7 Solve problems using the rules for indices and the rules for logarithms.

2.8 Demonstrate a fundamental understanding of binary numbers. Represent a number as a binary number. Perform binary addition. Convert from binary to base 10 and base 10 to binary.

2.9 Understand the concept of a complex number and illustrate their representation on an Argand diagram, be able to add, subtract and multiply complex numbers and calculate and interpret the modulus of a complex number.

3 Set Theory and Logic

The concept of 'set' is important in STEM disciplines.

The purpose of the outcomes in this unit is to recognise learners who can conceptualise sets and have the tools and skills required for exploring and expressing the relationships between sets. These include the Boolean logic skills required to analyse statements (propositions) and use equivalence of compound statements and test their validity in the context of practical applications.

3.1 Use the language of set theory appropriately including: universal set, subsets, sets N, Z, Q, R, C and ø, finite and infinite sets, and cardinal number of a set.

3.2 Explain the basic operations on sets including union, intersection, complement, symmetric difference, Cartesian product, and power set.

3.3 Use Venn diagrams of two and three sets to represent relationships between sets.

3.4 Define the Boolean operations AND, NOT, OR and XOR.

3.5 Define propositions/statements .

3.6 Define the truth tables for the compound statements AND, NOT, OR and XOR.

3.7 Use truth tables to establish logical equivalences for example De Morgan's Laws.

3.8 Explain the relationship between logical equivalences and set identities.

4 Algebra

Strong knowledge of, and skills in basic algebra and the ability to apply these skills to a range of problems is essential for the solution of many problems in STEM disciplines.

The purpose of the outcomes in this unit is to recognise learners who have an insight in to methods for the manipulation of algebraic expressions and are able to demonstrate ability, with mastery, to reliably manipulate algebraic expressions. Achievement (mastery) of the learning outcomes in this unit is essential for award of the L5 Maths for STEM qualification.

4.1 Distinguish between an expression and an equation.

4.2 Evaluate, expand and simplify algebraic expressions.

4.3 Transpose formulae and perform arithmetic operations on polynomials and rational algebraic expressions.

4.4 Multiply linear expressions to produce quadratics and cubics.

4.5 Reduce quadratic expressions to products of linear expressions through the use of inspection to determine the factors. Use this to solve quadratic equations.

4.6 Solve quadratic equations with real and complex roots by factorisation or formula. (see Functions 5.5) Solve cubic equations with at least one integer root.

4.7 Solve linear inequalities.

4.8 Find a solution, if it exists, for simultaneous linear equations with 2 and 3 unknowns and interpret the results.

5 Functions and Calculus

The mathematical notion of a function is important in STEM disciplines. This notion is not confined to real valued function of a real variable.

The purpose of the outcomes on this unit is to recognise learners who, in the special case of a real valued function of a real variable, have been introduced to the differential and integral calculus and are able to use these to investigate such functions and to show how real life problems of rates of change, areas and averages can be solved. Learners should not only be able to perform routine calculations, although mastering of these is an absolute requirement, but should also understand the theory, the power, and the limitations of the methods concerned.

5.1 Recognise that a function assigns a single output to every input, understand the concept of an inverse function and be able to compute it in simple algebraic cases.

5.2 Use mathematical modelling to investigate the relationship between two variables

5.3 Graph linear, quadratic, and cubic functions, and use these graphs to solve equations $f(x) = 0$, $f(x) = k$ and $f(x) = g(x)$.

5.4 Define and graph simple exponential, logarithmic, and trigonometric functions.

5.5 Understand growth/decay characteristics of different types of functions

5.5 Complete the square for a quadratic function and hence determine its roots and turning point. (see Algebra 4.5)

5.6 Investigate the concept of the limit of a function and compute the limits of linear, quadratic and quotient functions, and understand the idea of a continuous function.

5.7 Understand how a derivative arises as a limit from looking for tangent lines or rates of change.

5.8 Differentiate the following types of function: polynomial, trigonometric, rational power, exponential and logarithmic.

5.9 Use the sum, product and quotient formulas for differentiation and the chain rule to differentiate functions that are a composition of several functions.

5.10 Use derivatives to calculate tangent lines, rates of changes, maxima and minima, and whether functions are increasing or decreasing.

5.11 Understand that the definite integral of a positive function defines the area under a curve and that the Fundamental Theorem of Calculus reduces integration to finding anti-derivatives/indefinite integrals.

5.12 Be able to find the anti-derivative of polynomials, exponential, and trigonometric functions and linear combinations of these.

5.13 Be able to find the area under such positive curves.

5.14 Understand that a definite integral also gives the average of a function over an interval multiplied by the length of the interval and hence find average values.

6 Geometry and Trigonometry

Logical thought and deductive reasoning are key to STEM disciplines. Synthetic Geometry provides a mechanism for exploring logical thought and deductive reasoning. Through the proving of theorems learners will have the concept of a clear conclusion and the value of a clear proof.

The purpose of trigonometry and co-ordinate geometry is to provide learners with basic tools to solve problems in, and explore truths about, the physical world.

These theorems specified in the section on Synthetic Geometry should be known (proof not required for all) and the learner should be capable of applying them to solve practical problems across all areas of mathematics.

Synthetic Geometry

Know the statement of, and be able to solve problems using, the following theorems:

6.1 Theorem 1: Vertically opposite angles are equal in measure.

6.2 Theorem 2: Isosceles triangle: In an isosceles triangle the angles opposite the equal sides are equal. (ii) Conversely, if the two angles are equal, then the triangle is isosceles.

6.3 Theorem 3: Alternate angles: Suppose that A and D are on opposite sides of the line BC. If $|\angle ABC| = |\angle BCD|$, then AB $\parallel$ CD. In other words, if a transversal makes equal alternate angles on two lines, then the lines are parallel. Conversely, if AB $\parallel$ CD, then $|\angle ABC| = |\angle BCD|$. In other

words, if two lines are parallel, then any transversal will make equal alternate angles with them.

6.4 Theorem 4: The angles in any triangle add to 180 degrees.

6.5 Theorem 5: Corresponding Angles: Two lines are parallel if and only if for any transversal, corresponding angles are equal.

6.6 Theorem 6: Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

6.7 Theorem 7: (i) In Δ ABC, suppose that |AC| > |AB|. Then |$\angle$ ABC| > |$\angle$ ACB|. In other words, the angle opposite the greater of two sides is greater than the angle opposite the lesser side. (ii) Conversely, if |$\angle$ ABC| > |$\angle$ ACB|, then |AC| > |AB|. In other words, the side opposite the greater of two angles is greater than the side opposite the lesser angle.

6.8 Theorem 8: Two sides of a triangle are together greater than the third.

6.9 Theorem 9: In a parallelogram, opposite sides are equal, and opposite angles are equal.

6.10 Theorem 10: The diagonals of a parallelogram bisect each other.

6.11 Theorem 11: If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

6.12 Theorem 12: Let Δ ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s : t, then it also cuts [AC] in the same ratio. Know the proposition that if two triangles Δ ABC and Δ A'B'C' have |$\angle$ A| = |$\angle$ A'|, and |A'B'|/|AB| = |A'C'|/|AC|, then they are similar.

6.13 Theorem 13: If two triangles Δ ABC and Δ A'B'C' are similar, then their sides are proportional in order.

6.14 Theorem 14: Pythagoras: In a right angle triangle the square of the hypotenuse is the sum of the squares of the other two sides.

6.15 Theorem 15: Converse to Pythagoras: If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.

6.16 Theorem 16: For a triangle, base times height does not depend on the choice of base.

6.17 Theorem 17: A diagonal of a parallelogram bisects the area.

6.18 Theorem 18: The area of a parallelogram is the base by the height.

6.19 Theorem 19: The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

6.20 Theorem 20: (i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle s, and a line l through P is perpendicular to the radius to P, then l is tangent to s.

6.21 Theorem 21: The perpendicular from the centre of a circle to a chord bisects the chord.

6.22 Prove theorems 1, 3, 4, 12, 14.

Co-ordinate geometry

6.23 Work with linear equations ax + by + c =0.

6.24 Solve problems involving slope of a line to include investigating parallel and perpendicular lines.

6.25 Solve problems involving midpoint and length of a line segment.

6.26 Recognise that (x-h)^2 + (y-k)^2 = r^2 represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r.

Trigonometry

6.27 Understand the concepts of degree and radian measure.

6.28 Define sin θ, cos θ, tan θ, using right angled triangles and using the unit circle.

6.29 Work with trigonometric ratios in root form.

6.30 Solve problems involving the area of a triangle using the formula area = ½ab sin Θ

6.31 Solve practical problems using trigonometric formulae and terminology, including the sine, cosine and tangent ratios for right angled triangles.

6.32 Solve practical problems using the Sine Rule and Cosine Rule.

7 Probability and Statistics

Statistics is the science of data and statistical methods are underpinned by probability which is an important part of Maths. With the vast increase in the amount of data produced in all areas of STEM it is important that anyone pursuing further study or aiming to work in this field should be capable of analysing data.

The purpose of the outcomes in thus unit is to recognise learners who understand the basic concepts of probability and fundamental principles important in all data collection in STEM and who can apply basic methods for describing and evaluating data.

Counting

7.1 List outcomes of an experiment.

7.2 Apply the fundamental principle of counting (that if one event has m possible outcomes and a second independent event has n possible outcomes, then there are m x n total possible outcomes for the two events together).

7.3 Count the arrangements of n distinct objects (n!).

7.4 Count the number of ways of arranging r objects from n distinct objects.

7.5 Count the number of ways of selecting r objects from n distinct objects.

Probability

7.6 Recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur.

7.7 Engage in discussions about the purpose of probability.

7.8 Associate the probability of an event with its long run relative frequency .

7.9 Understand the concepts and be able to calculate probabilities by counting equally likely outcomes

7.10 Understand the concepts and be able to calculate compound probabilities of independent events and of mutually exclusive events

Statistical reasoning and data collection

7.11 Engage in discussions about the purpose of statistics and recognise misconceptions and misuses of statistics.

7.12 Discuss populations and samples.

7.13 Recognise the importance of representativeness so as to avoid biased samples and decide to what extent conclusions can be generalised from a sample to a population.

7.14 Understand how to select a sample using Simple Random Sampling.

7.15 Understand that randomness and representativeness are not the same.

7.16 Recognise that not every sample is the same and that different samples may lead to different estimates about a given population – this concept is known as sampling variability.

7.17 Discuss different types of studies: sample surveys, observational studies and designed experiments.

7.18 Design a plan and collect data on the basis of above knowledge.

Describing data graphically and numerically

7.19 Understand the different types of data: categorical: nominal or ordinal numerical: discrete or continuous.

7.20 Discuss the effectiveness of different displays in representing the findings of a statistical investigation (pie charts, histograms, stem and leaf plots).

7.21 Use histograms (equal intervals) to display data.

7.22 Understand and be able to compute: -

- mean, median, mode to measure central tendency;
- range and standard deviation (use a calculator to calculate standard deviation) to measure variability.

7.23 By reference to histograms, describe a distribution of data in terms of symmetry and skewness.

7.24 Discuss the limitations or merits of mean, median and mode for measuring central tendency with symmetric data and with skewed data.

7.25 Understand what bivariate data is and determine the relationship between variables using scatterplots.

Assessment

General Information

Details of FET assessment requirements are set out in Assessment Guidelines for Providers.

All FET assessment is criterion referenced. Successful achievement of the award is based on learners attaining the required standards of knowledge, skill or competence.

The techniques set out below are considered the optimum approach to assessment for this component. In exceptional circumstances providers may identify alternative assessment techniques through the provider's application for programme validation which are **reliable** and **valid** but which are more appropriate to their context.

Assessment of a number of components may be integrated across programmes for delivery, provided that the learning outcomes of each minor award are assessed.

Group or team work may form part of the assessment, provided each learner's achievement is separately assessed.

All providers are required to submit an assessment plan as part of their application for programme validation. Assessment Plans will include information relating to scheduling and integration of assessment. See current FET validation guidelines at www.qqi.ie.

Assessment Techniques

In order to demonstrate that they have reached the standards of knowledge, skill and competence identified in all the learning outcomes, learners are required to complete the assessment(s) below.
The assessor is responsible for devising assessment instruments (e.g. project and assignment briefs, examination papers), assessment criteria and mark sheets, consistent with the techniques identified below and QQI's assessment requirements.
Programme validation will require providers to map each learning outcome to its associated assessment technique. See current FET validation guidelines at www.qqi.ie.
All learning outcomes must be assessed and achieved

Continuous Assessment	Weighting 30%
Final Examination	Weighting 50 %
Multiple Choice Questions	Weighting 20 %

Description

Assessment Model
This is indicative of the assessment to be used. The assessment specifications will reviewed and confirmed after a pilot programme has been developed and a set of sample assessments developed.

Assessment technique(s) including weighting(s)

In order to demonstrate that they have reached the standards of knowledge, skill and competence identified in all the learning outcomes learners are required to complete the assessment(s) below.

Continuous Assessment (proctored): 30%

2 X 15% proctored assessment. One assessment from Unit 7 and one assessment from Unit 3, 5 or 6.

For example: a tutorial sheet could consist of questions for each unit. Students are given a suitably large bank of tutorial questions to do outside of class time. Summative assessment is given under supervised conditions using questions from the bank of tutorial questions.

Final Examination: 50%

2 X 25% examinations.

Examination Paper 1: 10 short answer questions from Unit 3 and Unit 6 (1% each question); 3 long answer questions testing the material from Unit 2, Unit 3 and Unit 6 (5% each question).

Examination Paper 2: 10 short answer questions from Unit 5 and Unit 7 (1% each question); 3 long answer questions testing the material from Unit 4, Unit 5 and Unit 7 (5% each question).

Multiple Choice Questions (MCQ): 20%

MCQ on Units 2 and 4 only. MCQ is of one hour's duration with a pass threshold of 80%. Pass in the MCQ is mandatory for overall pass of this award.

Recognition of Prior Learning (RPL)

Learners may be assessed on the basis of their prior knowledge and experience. Providers must be specifically quality assured to assess learners by this means. To do so they must complete B10, see Provider's Quality Assurance Guidelines and be included on the Register of RPL approved providers. See RPL Guidelines at www.qqi.ie for further information and registration details.

Grading

Pass	50% - 64%
Merit	65% - 79%
Distinction	80% - 100%

Specific Validation Requirements

None

Supporting Documentation

Providers should take cognisance of relevant research and legislation in designing programme content

Access

To access programmes leading to this award the learner should have reached the standards of knowledge, skill an competence associated with the preceding level of the National Framework of Qualifications. This may have been achieved through formal qualification or through relevant life and work experience.

For the purpose of assigning credits to programmes leading to this award an entry standard equivalent to the Level 4 Minor Award in Mathematics is assumed

Transfer

Successful completion of this component award enables the learner to transfer to programmes leading to other certificates where this component is a mandatory or an elective requirement.

2. FET Award Standards

QQI award standards are determined within the National Framework of Qualifications (NFQ), http://www.nfq-qqi.com. QQI determines standards for the education and training awards that it makes itself and that are made by providers to whom it has delegated authority to make an award. Providers offering programmes leading to QQI awards **must** have their programme(s) validated in accordance with current validation policy (see www.qqi.ie).

Award standards are designed to be consistent with the NFQ's award classes i.e. major, special purpose, supplemental and minor awards. They are expressed in terms of **learning outcomes** i.e. concise statements of what the learner is expected to know or be able to do in order to achieve a particular award. Learning outcomes for FET awards are contained within the associated specifications:

AWARD CLASS	STANDARDS	AWARDS
Major Award	Certificate Specification	Certificate (Levels 1 to 5) Advanced Certificate (Level 6)
Supplemental Award	Supplemental Specification	Supplemental Certificate (Level 3 to 6)
Special Purpose	Specific Purpose Specification	Specific Purpose Certificate (Levels 3 to 6)
Minor Award	Component Specification	Component Certificate (Levels 1 to 6)

Award standards are thresholds, they describe standards of knowledge, skill or competence to be acquired, and where appropriate, demonstrated, by a learner before an award may be made.

Award standards will be reviewed from time to time as necessary. Minor changes may be made by the QQI executive outside the review cycle where necessary. Changes to standards are published on QQI's website. Providers with validated programmes and providers with delegated authority to make awards are responsible for monitoring relevant standards and making necessary responses to changes.

3. FET Credit

Every FET certificate and component specification includes an FET credit value (Table 1). FET credit is quantified in multiples of 5 FET credits (up to 50 hours of learner effort). Learner effort is based on the time taken by typical learners at the level of the award to achieve the learning outcomes for the award. It includes all learning time involved including: guided learning hours, self-directed learning and assessment.

Table 1: FET Credit Values

NFQ Level	Major Awards Credit Values	Default Credit Values Minor Awards	Other Permitted Minor Award Credit Values	Special Purpose and Supplemental Award Credit Value Ranges
1	20	5	10	
2	30	5	10	
3	60	10	5,20	>5 and<60
4	90	10	5,15,20	>5 and<90
5	120	15	5,10,30	>5 and <120
6	120	15	5,10,30	>5 and <120

Guide to Level

Learning outcomes at this level include a comprehensive range of skills which may be vocationally-specific and/or of a general supervisory nature, and require detailed theoretical understanding. The outcomes also provide for a particular focus on learning skills. The outcomes relate to working in a generally autonomous way to assume design and/or management and/or administrative responsibilities. Occupations at this level would include higher craft, junior technician and supervisor.

Strand	Sub-strand	Nature of learning
Knowledge	Breadth	Specialised knowledge of a broad area
	Kind	Some theoretical concepts and abstract thinking, with significant depth in some areas
Know How & Skill	Range	Demonstrate a comprehensive range of specialised skills and tools
	Selectivity	Formulate responses to well defined abstract problems
Competence	Context	Utilise diagnostic and creative skills in a range of functions in a wide variety of contexts
	Role	Exercise substantial personal autonomy and often take responsibility for the work of others and/or for the allocation of resources; form and function within, multiple and complex heterogeneous groups.
	Learning to Learn	Learn to take responsibility for own learning within a managed environment.
	Insight	Express an internalised, personal world view, reflecting engagement with others.

Extract from 'Determinations for the Outline National Framework of Qualifications': NQAI